Sketching quadratics

It is not always necessary in questions, but sketching quadratics can help identify solutions, or help in the interpretation of questions in context.
A quadratic equation has the basic form $y = ax^2 + bx + c$, but it can also be expressed with the square completed, $y = A(x - B)^2 + C$, and *sometimes* in factorised form, $y = (Ax - B)(Cx - D)$, where A, B, C and D are constants.
Rather than finding *x*,*y* pairs for a range of values of *x* and drawing a curve through those points as you might do when a more accurate graph is required, sketching just involves indicating the shape of the curve, and the labelling of easily found, but key features of the quadratic, such as:

- maximum or minimum point
- *y*-intercept
- *x*-intercept(s), if any.

Examples

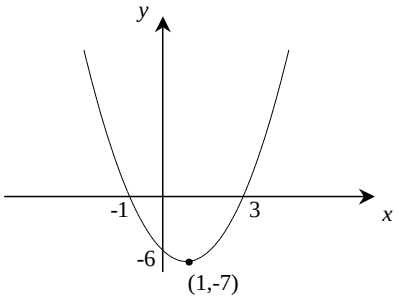
Sketch the graph of the quadratic function $y = x^2 - 2x - 6$.

The graph cuts the *y*-axis ($x = 0$) at $y = -6$.

$y = x^2 - 2x - 6 = (x - 3)(x + 1)$
So, graph cuts the *x*-axis ($y = 0$) when $x = 3$ and when $x = -1$

Also, $y = x^2 - 2x - 6 = (x - 1)^2 - 7$.
Graph has a minimum at (1,−7), because the term $(x - 1)^2 \geq 0$ for all possible *x* values, and is minimum, i.e. 0, when x = 1.

Sketch of quadratic:



Notes

The *y*-intercept is found by putting $x = 0$ (and is just the constant term for a quadratic function).

Quadratics that can be simply factorised can quickly give the points where the curve cuts the *x*-axis. (*Not all quadratics can be factorised in this way*.)

All quadratics can be written in the square completed form. This gives the maximum/minimum of a quadratic function.

Sketching quadratic curves takes practice!

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Examples

Sketch the graph of the quadratic function
$y = -2x^2 + 8x + 1$.

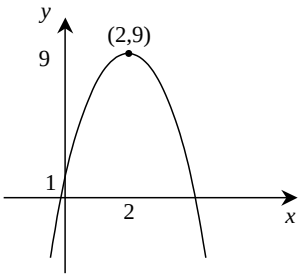
The graph cuts the y-axis at $y = 1$.

The quadratic does not factorise simply.

$$y = -2x^2 + 8x + 1$$
$$y = -2(x^2 - 4x) + 1$$
$$y = -2\left((x-2)^2 - 4\right) + 1$$
$$y = -2(x-2)^2 + 8 + 1$$
$$y = -2(x-2)^2 + 9$$

Graph has a maximum at (2, 9), because the term $-2(x-2)^2 \le 0$ for all possible x values, and is maximum when the term is equal to 0, when $x = 2$

Sketch of quadratic:



Notes

y-intercept found when $x = 0$

You can check this by using the quadratic formula that it has solutions involving irrational numbers.

Complete the square.
Be careful when taking a negative number out as a factor!

From finding just the maximum point and the y-intercept and sketching the curve, you can see that the quadratic equation has one negative solution quite close to zero and one positive solution greater than 4 (by symmetry of the quadratic about the line x=2 through the maximum point.)

Some to try

Find the y-intercept and maximum (or minimum) points of the following quadratics by completing the square. Then sketch their graphs, labelling these points.

1	$y = x^2 + 2x$	**2**	$y = x^2 - 4x + 1$	**3**	$y = x^2 - 6x + 10$	**4**	$y = -x^2 + x + 1$
5	$y = x^2 - 5x - 6$	**6**	$y = x^2 - x + 3$	**7**	$y = x - x^2$	**8**	$y = -x^2 - 2x - 1$
9	$y = 2x^2 - 4x + 3$	**10**	$y = 3x^2 - 3x + 1$	**11**	$y = 5 - 2x^2 + 8x$	**12**	$y = x - 5x^2 - 2$

By factorising the following quadratics, find, in addition to the y-intercept and the maximum (or minimum) points, the x-intercept(s) and then label all these points on a sketch of the quadratic curve.

13	$y = x^2 - 2x + 1$	**14**	$y = x^2 + 5x + 6$	**15**	$y = x^2 + 4x - 5$	**16**	$y = x^2 - x - 12$
17	$y = x^2 + 6x + 8$	**18**	$y = x^2 - 10x + 16$	**19**	$y = -2x^2 - 5x + 3$	**20**	$y = 2x^2 - 9x - 5$
21	$y = 2 + x - 3x^2$	**22**	$y = 4x^2 + 4x + 1$	**23**	$y = 4x^2 - 2x - 12$	**24**	$y = 12x^2 + 20x + 3$

Some of the following can be factorised, others cannot. Without resorting to using the quadratic formula to find the solutions, label as fully as you can, sketches of the following quadratics.

25	$y = x - x^2$	**26**	$y = x^2 + 3x + 3$	**27**	$y = x^2 - 2x - 1$	**28**	$y = x^2 - 1$
29	$y = -2 - x - x^2$	**30**	$y = 2x^2 - 6x - 8$	**31**	$y = 3x^2 - 12x - 10$	**32**	$y = x^2 - 2x - 15$
33	$y = 5x^2 - 2x + 3$	**34**	$y = 2x(2 - x) + 3$	**35**	$y = 2x - x^2 - 1$	**36**	$y = x^2 + 5x - 6$

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Answers

For 1–12, the *y*-intercept is followed by the co-ordinates of the minimum or maximum point.

1	0, min (-1,-1)	**2**	1, min (2,-3)	**3**	10, min (3,1)
4	1, max (0.5, 1.25)	**5**	–6, min (2.5, -12.25)	**6**	3, min (0.5, 2.75)
7	0, max (0.5, 0.25)	**8**	–1, max (-1,0)	**9**	3, min (1,1)
10	1, min (0.5, 0.25)	**11**	5, max (2,13)	**12**	–2, max (0.1, -1.95)

For 13–24, the *y*-intercept, the co-ordinates of the minimum or maximum point, and the *x*-intercept(s) are given in order, separated by a semi-colon.

13	1; min (1,0); 1	**14**	6; min (-2.5, -0.25); -2, -3
15	–5; min (-2,-9); -5, 1	**16**	–12; min (0.5, -12.25); -3, 4
17	8, min (-3,-1); -2, -4	**18**	16; min (5,-9); 2, 8
19	3, max (-1.25,6.125); -3, ½	**20**	–5; min (2.25, -15.125); -½, 5
21	2, max (1/6, 25/12); -2/3, 1	**22**	1, min (-½, 0); -½
23	–12 min (0.25, -12.25); -3/2, 2	**24**	3, min (-5/6, -16/3); -1/6, -3/2

For 25–36, the *y*-intercept, the co-ordinates of the minimum or maximum point, and the *x*-intercept(s) (where they exist and can be found by factorisation), are given in order, separated by a semi-colon.

25 0; max (0.5, 0.25); 0, 1

26 3; min (-1.5, 0.75); doesn't cross *x*-axis

27 –1; min (1, -2); not easily factorised

28 –1; min (0, -1); -1, 1

29 –2; max (-0.5, -1.75); doesn't cross *x*-axis

30 –8; min (1.5, -12.5); -1, 4

31 10; min (2, -22); not easily factorised

32 –15; min (1, -16); -3, 5

33 3; min (0.2, 2.8); doesn't cross *x*-axis

34 3; max (1, 5); not easily factorised

35 –1; max (1, 0); only at 1

36 –6; min (-2.5, -12.25); -6, 1

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